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Renormalization group analysis of nuclear force

S.X. Nakamura[a]

[a] Theory Group, TRIUMF, 4004 Wesbrook Mall, Vancouver, BC V6T 2A3, Canada

We study a relation between nuclear forces based on phenomenological approach (V_{ph}) and nuclear effective field theory (V_{EFT}) from a viewpoint of renormalization group. We find the relation between these two types of nuclear force using Wilsonian renormalization group equation. Considering the fact that V_{EFT} is defined in a certain small model space, we show that a simple contact interaction accurately simulates short-distance physics, and that V_{EFT} is free from dependence on modelling the details of the short-distance physics. Based on our result, we discuss some features of nuclear effective field theory.

1. Introduction

Nuclear force is one of the oldest problem in nuclear physics. However, its theoretical description is still an unsettled issue. Here, we first explain a traditional, phenomenological approach to the nuclear force, and a fairly new approach based on nuclear effective field theory (NEFT)[1]. After the explanation, we formulate a scenario for a relation between nuclear forces based on the two approaches. The main subject in this report is to discuss the relation from a viewpoint of the renormalization group (RG).

In the phenomenological nuclear force (V_{ph}), the long-range part is described by the one-pion-exchange potential (OPEP). The short-range part is poorly known, and therefore a phenomenological model is used; heavy-meson exchanges or purely phenomenological parameterization. Parameters involved in the model are fixed so as to reproduce low-energy NN scattering data and the deuteron binding energy. In this way, several high-precision NN potentials have been constructed, such as the CD-Bonn and the Nijmegen potentials. Although these models are phenomenologically very successful, there are some problems as follows. At first, each model describes the short-range part differently, and is largely model-dependent. Secondly, there is no systematic way to construct the short-range mechanism. Thirdly, there is little connection to the underlying theory, QCD.

In contrast, it is claimed that a nuclear force based on NEFT (V_{EFT}) is free from the problems inherent in V_{ph}. The claim is based on the derivation procedure of V_{EFT}, which is briefly given as follows. One starts with an effective chiral Lagrangian, taking care of the spontaneously broken chiral symmetry of QCD. The Lagrangian consists of effective degrees of freedom for a system in question, and is the most general as long as assumed symmetries are satisfied. One identifies a set of irreducible diagrams from the Lagrangian with V_{EFT}. The importance of an irreducible diagram is assigned by a counting rule.

However, one may find questions about V_{EFT}: (Q1) In V_{EFT}, short-distance physics is described by contact interactions, which is much simpler compared to the short-range

mechanism employed in V_{ph}. Is the contact interaction really appropriate to describe the short-distance physics? (Q2) Is V_{EFT} still one of many phase-equivalent potentials, as V_{ph} is? (Q3) Is there any relation between model independent V_{EFT} and model dependent V_{ph}? In fact, the answer to (Q3) naturally leads us to the answers to (Q1) and (Q2). Thus, we formulate a scenario for the relation between V_{EFT} and V_{ph}.

It is noted that the model space (the state space for the nucleon) of V_{EFT} is considerably smaller than that of V_{ph}. With this point in mind, we can formulate the following scenario for the relation between V_{EFT} and V_{ph}. We may construct many V_{ph} which reproduce NN scattering data and the deuteron binding energy. They are different in describing the short-distance physics, and therefore model dependent. Starting with such V_{ph}, we reduce their model space by integrating out the high momentum states of the nucleon. Reducing the model space corresponds to viewing the system in a coarse-grained manner. As the model space is reduced, information about details of the short-distance physics is gradually lost. Eventually, we obtain a low-momentum effective interaction (V_M) defined in a reduced model space. V_M does not have the model dependence which V_{ph} have. A parameterization of V_M constitutes V_{EFT}. This is the scenario for the relation between V_{EFT} and V_{ph}. V_{EFT} obtained in this way is, by construction, does not have a dependence on modelling the short-distance physics. The short-range part of V_M is expected to be accurately simulated by simple contact interactions because the detailed information has been integrated out. Therefore, if we show that this scenario is realized, then we can answer all of the questions raised in the previous paragraph.

The purpose of this report is to confirm the scenario, thereby proposing the relation between V_{EFT} and V_{ph}[2]. Furthermore, keeping the relation in mind, we can understand features of NEFT more deeply, which we will discuss later.

2. Wilsonian renormalization group equation

In order to examine the scenario, stated in the previous section, for the relation between V_{EFT} and V_{ph}, an appropriate model-space reduction method is necessary. In effective filed theory, we use an effective Lagrangian in which high-energy degrees of freedom have been integrated out using a path integral:

$$Z = \int \mathcal{D}\Psi_H \mathcal{D}\Psi_L \, e^{i\int d^4x \ \mathcal{L}_H} = \int \mathcal{D}\Psi_L \, e^{i\int d^4x \ \mathcal{L}_L} \ , \tag{1}$$

where Ψ_H (Ψ_L) is high-(low-) energy degrees of freedom, and $\mathcal{L}_H$ ($\mathcal{L}_L$) consists of Ψ_H and Ψ_L (only Ψ_L). An effective (more fundamental) Lagrangian is $\mathcal{L}_L$ ($\mathcal{L}_H$). The model-space reduction, integrating out high-momentum states of the nucleon, also should be done with the path integral. Although it is difficult in general, in case the Lagrangian is composed by only the nucleon field and two nucleons interact through contact interactions, we can perform the path integral. It is noted that we restrict ourselves to two-nucleon system throughout this report. In the center of mass frame, performing the path integral is equivalent to solving the following Wilsonian renormalization group (WRG) equation:

$$\frac{\partial V^{(\alpha)}(k',k;p,\Lambda)}{\partial \Lambda} = \frac{M}{2\pi^2} V^{(\alpha)}(k',\Lambda;p,\Lambda) \frac{\Lambda^2}{\Lambda^2 - p^2} V^{(\alpha)}(\Lambda,k;p,\Lambda) \ , \tag{2}$$

where $V^{(\alpha)}$ is NN potential for a partial wave α, and M the nucleon mass. In $V^{(\alpha)}(k', k; p, \Lambda)$, k (k') denotes the off-shell relative momentum of the two nucleons before (after) the interaction, p denotes the on-shell momentum, and Λ is the cutoff value specifying the model space. This equation controls the evolution of $V^{(\alpha)}$ with respect to a change of Λ. This equation was firstly derived by Birse *et al.* in a different manner[3].

3. Result

Starting with several phenomenological NN potentials, we reduce their model spaces using the WRG equation, and examine the evolution. Then, we simulate the obtained effective model-space interaction (V_M) with V_{EFT} by adjusting parameters in it. If the simulation is accurate, then the relation between V_{EFT} and V_{ph} through RG is realized.

In the simulation, for simplicity, we include only the OPEP in V_{EFT} as a mechanism explicitly including the pion. This is not fully consistent with Weinberg's counting. Because we consider contact interactions with zero, two, and four derivatives, we should include more irreducible graphs, such as a two-pion exchange potential (TPEP). However, we employ a rather small model space where the details of the TPEP play essentially no role; the TPEP is accurately simulated by contact interactions. Therefore, our simplification does not deteriorate the accuracy of the simulation.

In Fig. 1, shown are diagonal components of $\langle {}^1S_0|V|{}^1S_0\rangle$ in the momentum space. There is the clear model dependence among V_{ph}, which almost disappears after the model-space reduction down to $\Lambda = 200$ MeV. The obtained V_M is accurately simulated by the OPEP and a few contact interactions, which we do not show here. In Fig. 2, diagonal components of $\langle {}^3D_1|V_M|{}^3S_1\rangle$ (Λ=200MeV) obtained from the CD-Bonn potential is simulated using the OPEP and one or two contact interactions. We see that the simulation is very accurate. V_{EFT} obtained in this way also accurately simulates the off-diagonal components of V_M.

4. Discussion and summary

We have seen in the previous section that the scenario for the relation between V_{EFT} and V_{ph} is realized. Although we have shown the relation employing a rather small model space, if the TPEP and higher order OPEP are properly included in V_{EFT}, then the relation should hold true for a larger model space as well; probably safe up to $\Lambda \sim 400$ MeV. In a certain large model space where the very details of the TPEP is considerable, *i.e.*, a phenomenological model cannot mimic the TPEP any more, the relation may not hold true. We have been concerned with only the strong nucleon-nucleon interaction. However, this kind of relation probably exists also in other nuclear operators, such as electroweak currents and pion production operators.

Now, based on our result, we discuss in the following some features of NEFT which the traditional approach does not have; the discussion is not always restricted to the nuclear force. One is that nuclear operators based on NEFT are constructed perturbatively. Therefore, one can systematically improve the accuracy, and can discuss theoretical uncertainty. We note that, for a convergent perturbation, it is necessary to define an operator in a suitably small model space. We also note that the convergence of the NEFT-based perturbative expansion is not always good for some processes.

Another is that NEFT-based operators are model independent, which has been often

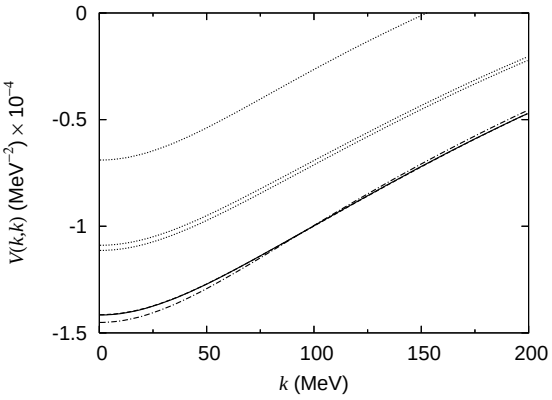

Figure 1. NN-potential $\langle {}^1S_0|V|{}^1S_0\rangle$; the upper, the Reid93, the Nijmegen I and the CD-Bonn potentials, respectively. The solid and dash-dotted curves are V_M ($\Lambda =$ 200 MeV), the solid one results from the Nijmegen I and the CD-Bonn potentials, while the dash-dotted from the Reid93.

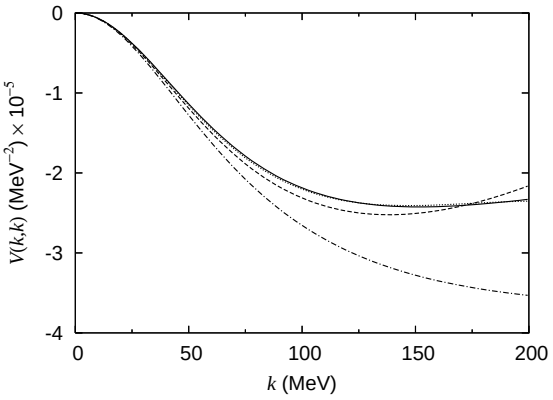

Figure 2. NN-potential $\langle {}^3D_1|V|{}^3S_1\rangle$. The solid curve is $V_M(\Lambda$=200MeV) resulting from the CD-Bonn potential. The dashed (dotted) curve is a simulation of V_M using the OPEP plus one (two) contact interaction(s). The dash-dotted curve is the OPEP.

claimed. However, the claim is based on the procedure of deriving the nuclear operators, which is not a quantitative argument. In this work, we quantitatively showed that V_{EFT} does not have the dependence on modelling the details of short-distance physics. This is a consequence of viewing the system roughly, thereby ignoring the detailed structure of the short-distance physics. It is noted, therefore, a description of a system based on NEFT is not always better than those based on phenomenological models. In many cases, they give essentially the same low-energy observables because they are equivalent through the RG. NEFT may have superiority in cases where detailed information about multi-pion exchange mechanism play an important role.

Finally, with NEFT, one can describe a system more efficiently and simply, compared to the phenomenological models. The efficiency is due to the perturbative calculation, and the simplicity is due to the ignorance of the details of the short-distance physics.

To summarize, we considered the nuclear force from the viewpoint of the RG, and showed that there exists the equivalence relation between V_{EFT} and V_{ph}. We simultaneously showed that V_{EFT} does not have the model dependence due to the details of the short-distance physics, and that the simple contact interactions in V_{EFT} accurately describe the short-distance physics in a certain small model space. Based on the result, we discussed the new features in NEFT.

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